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CALCULATIONS OF THE EFFECTIVE AND APPARENT JOULE–THOMSON COEFFICIENTS UPON GASSY LIQUID FILTRATION

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The equations of a continuous medium upon filtration of a gassy liquid have been analyzed, and on this basis the equation of energy with effective coefficients has been obtained. Analytical expressions have been constructed and calculations of the effective and apparent Joule–Thomson coefficients and the effective convective heat-transfer velocities and heat capacity per unit volume of the porous medium in the degassing zones and in the free gas zones have been performed. The laws of change in the above quantities depending on the pressure and the radial coordinate have been investigated.

The problem on the temperature fields upon gassy liquid filtration is one of the major problems in describing oil motion in underground oil traps. This is due to the fact that oils of the majority of oil fields contain a considerable amount of gas, mainly methane. The necessary condition for solving the temperature problem upon gassy liquid filtration is the finding of the effective parameters entering into the energy equation, since the medium being filtered consists of several phases.

The aim of this work is to construct analytical expressions for the effective and apparent Joule–Thomson coefficients and make calculations on their basis.

In a number of oil fields, the saturation pressure is equal to the pool pressure and oil motion in the pool is accompanied by degassing throughout [1]. Under such conditions, the temperature decreases due to both the heat absorption at the phase transition and the Joule–Thomson effect in the gas phase. The liquid-phase flow leads to an increase in the temperature, and the Joule–Thomson coefficient is positive in this case. For small gas solubilities a heating of the pool is observed and for large ones there is a cooling of the gas-liquid flow.

But if the saturation pressure is lower than the pressure in the pool, then two cases of flow can be realized: when the pressure in the well is higher than or equal to the saturation pressure, there is a one-phase flow accompanied by heating due to the barothermal effect (or the Joule–Thomson effect) in the oil, and when the saturation pressure is in the interval between the pool and well pressures, then a two-zone flow is realized. In the zone where the pressure exceeds the saturation pressure, $P_s < P < P_c$, a one-phase flow of oil accompanied by heating takes place. In the zone where the pressure is lower than the saturation pressure $P_w < P < P_s$, there is an oil and gas flow accompanied by phase transitions and the barothermal effect, which leads, for large solubility coefficients, to a cooling of the flow.

From the theoretical point of view, the two-zone flow seems to be the most general, since the other cases are particular or limiting. For this reason we give primary consideration to the two-zone flow. The physical characteristics of the oil and gas traps vary in both the pool thickness and the strike, i.e., the vertical and horizontal coordinates. This means that in the general case the oil-gas pools are inhomogeneous (and anisotropic). For the sake of simplicity, we assume that a porous pool is homogeneous in hydrodynamic and thermophysical properties.

Under these assumptions, Fig. 1 illustrates the geometry of the problem under consideration, where gassy oil moves towards a well of radius r_w. In this problem, it is convenient to use a cylindrical system of coordinates. It is assumed that the problem has an axial symmetry, i.e., the hydrodynamic and thermophysical fields and the physical parameters are independent of the angular coordinate φ.

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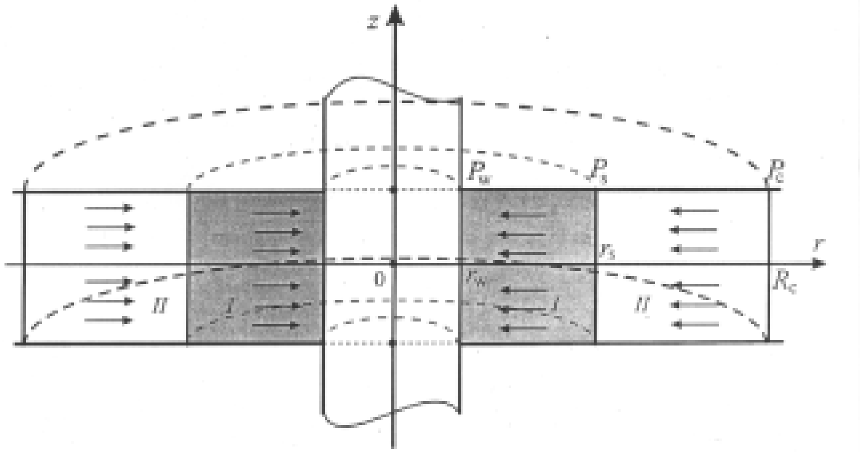

Fig. 1. Gassy oil filtration in an infinite pool; geometry of the problem.

The well pressure P_w is lower than the saturation pressure P_s reached at $r = r_s$ in the pool. Therefore, in the region of $r_w < r < r_s$ a two-phase flow of oil and gas is realized (in Fig. 1, this region is darkened, zone I). In the region from the saturation radius r_s to the feed contour radius R_c, a one-phase flow of oil is realized (zone II in Fig. 1).

The mathematical formulation of this problem is described by the mass and angular momentum equations, the equation of state of the free gas, and by the Henry law. The mass equations are given in the form of continuity equations for the carrier phase, for the gas dissolved in the liquid, and for the free gas:

$$\frac{\partial (ms_1 \rho_1)}{\partial t} + \frac{1}{r}\frac{\partial (r\rho_1 V_1)}{\partial r} = 0\,, \quad \frac{\partial (ms_2 \rho_2)}{\partial t} + \frac{1}{r}\frac{\partial (r\rho_2 V_2)}{\partial r} = -q\,, \quad \frac{\partial (ms_3 \rho_3)}{\partial t} + \frac{1}{r}\frac{\partial (r\rho_3 V_3)}{\partial r} = q\,,$$

$$s_1 = s_2\,, \quad s_1 + s_3 = 1\,,$$

where subscripts $i = 1, 2, 3$ pertain to the carrier phase and the dissolved and free gas, respectively.

For the filtration model, the Darcy law is used. For the carrier phase and the free gas, with the capillary pressure jump neglected, it is written in the form

$$V_1 = -\frac{k}{\mu_1} f_1(s) \frac{\partial P}{\partial r}\,, \quad V_3 = -\frac{k}{\mu_3} f_3(s) \frac{\partial P}{\partial r}\,.$$

The carrier phase is assumed to be incompressible, and the equation of state of the free gas in the form of the Van der Waals equation

$$\left(P + \frac{a\rho_3^2}{M^2}\right)\left(\frac{M}{\rho_3} - b\right) = RT\,.$$

is used.

According to the Henry law [2], the free gas concentration is proportional to the pressure P:

$$\frac{\rho_2}{\rho_2 + \rho_1} = \alpha P\,.$$

It is assumed that at the initial time the pressure in the well instantaneously decreases to P_w and is further held constant $P = P_w$ ($r = r_w$, $t > 0$). At a distant boundary of the porous medium the condition $P = P_c$ ($r = R_c$, $t > 0$) is assumed. At time $t > 0$ at a distance of the saturation radius $r = r_s$ liberation of the free gas from the carrier phase begins.

One-Temperature Energy Equation for the Gas-Liquid Flow. Below, we give the energy equations for a carrier phase consisting of two components (oil and dissolved gas), free gas (second phase), and the porous medium skeleton, which are then transformed to one equation in the one-temperature approximation [3–6].

The energy equation for the skeleton

$$c_0\,\rho_0\,(1-m)\,\frac{\partial T_0}{\partial t} = \mathrm{div}\,[(\lambda_0\,(1-m))\;\mathrm{grad}\;T_0] - \sum_j \theta^*_{0j}\,(T_0 - T_j)$$

differs from the ordinary heat-conduction equation by the presence of a term taking into account the heat exchange with the saturating phases (the last term).

The energy equation for the absorbing phases is written with a subscript i:

$$c_i\,\rho_i s_i m\,\frac{\partial T_i}{\partial t} + \mathbf{V}_i c_i\,\rho_i\,[\mathrm{grad}\;T_i + \varepsilon_i\;\mathrm{grad}\;P] - m s_i\,\rho_i c_i \eta_i\,\frac{\partial P}{\partial t} + W_i q_i = \mathrm{div}\,[(m s_i \lambda_i)\;\mathrm{grad}\;T_i] - \sum_{j\neq i} \theta^*_{ij}\,(T_i - T_j)\,.$$

Summing the energy equation over all phases and components and assuming their temperatures equal, we obtain

$$\left[\sum_i m s_i\,\rho_i c_i + (1-m)\,\rho_0 c_0\right]\frac{\partial T}{\partial t} + \sum_i \mathbf{V}_i\,\rho_i c_i\,(\mathrm{grad}\;T + \varepsilon_i\;\mathrm{grad}\;P) -$$

$$-\sum_i m s_i\,\rho_i c_i \eta_i\,\frac{\partial P}{\partial t} + \sum_i L_i q_i = \mathrm{div}\left[\left(\sum_i m s_i \lambda_i + (1-m)\,\lambda_0\right)\mathrm{grad}\;T\right]. \tag{1}$$

Dividing Eq. (1) by the heat capacity per unit volume of the saturated porous medium $c_\mathrm{p} = \sum_i m s_i \rho_i c_i + (1-m)\rho_0 c_0$, we have

$$\frac{\partial T}{\partial t} + \frac{\sum_i \mathbf{V}_i\,\rho_i c_i}{c_\mathrm{p}}\,\nabla T + \frac{\sum_i \mathbf{V}_i\,\rho_i c_i \varepsilon_i}{c_\mathrm{p}}\,\nabla P - \frac{\sum_i m s_i\,\rho_i c_i \eta_i}{c_\mathrm{p}}\,\frac{\partial P}{\partial t} + \frac{Lq}{c_\mathrm{p}} = \frac{\mathrm{div}\left[\left(\sum_i m s_i \lambda_i + (1-m)\,\lambda_0\right)\mathrm{grad}\;T\right]}{c_\mathrm{p}}\,. \tag{2}$$

Expression (2) can be given in a form coinciding with the equation for the one-phase flow with the effective parameters

$$\frac{\partial T}{\partial t} + \mathbf{U}_\mathrm{ef}\,(\nabla T + \varepsilon_\mathrm{ef}\nabla P) - \eta_\mathrm{ef}\Pi\,\frac{\partial P}{\partial t} + \frac{Lq}{c_\mathrm{p}} = a_\mathrm{p}\Delta T\,, \tag{3}$$

where $\Pi = m \sum_i s_i \rho_i c_i / c_\mathrm{p}$; $a_\mathrm{p} = \sum_i s_i \lambda_i + (1-m)\lambda_0 / c_\mathrm{p}$; ε_ef is the effective Joule–Thomson coefficient; $\mathbf{U}_\mathrm{ef}$ is the effective convective heat-transfer velocity; η_ef is the effective adiabatic coefficient.

Effective Convective Heat-Transfer Velocity. Equating the coefficients at the temperature gradient in Eqs. (2) and (3), we obtain the expression for the effective convective heat-transfer velocity for the one-phase flow:

$$\mathbf{U}_\mathrm{ef} = \sum_i \mathbf{V}_i c_i\,\rho_i / c_\mathrm{p}\,. \tag{4}$$

In a homogeneous isotropic medium, the vectors $\mathbf{U}_\mathrm{ef}$ and $\mathbf{V}_i$ are collinear. Let us show this. It is known that the speeds of filtration of phases for a homogeneous isotropic medium obey the Darcy law, i.e., coincide in the direction with the pressure gradient

$$\mathbf{V}_i = -\frac{k}{\mu_i} f_i (s)\, \nabla P = -\frac{k}{\mu_i} f_i (s)\, |\nabla P|\, \mathbf{n} = - V_i\, \mathbf{n}\,, \tag{5}$$

$$V_i = \frac{k}{\mu_i} f_i (s)\, |\nabla P|\,, \tag{6}$$

where **n** is the unit vector in the direction of the pressure gradient.

From (5) it follows that all vectors $\mathbf{V}_i$ are collinear. Substituting the expression for the speed of filtration of the *i*th phase (*i*th component) (5) into (4), we obtain the expression

$$\mathbf{U}_{\text{ef}} = -\frac{k}{c_{\text{p}}} \sum_i \frac{c_i\, \rho_i\, f_i (s)}{\mu_i}\, |\nabla P|\, \mathbf{n} = - U_{\text{ef}}\, \mathbf{n}\,. \tag{7}$$

From (5) and (7) it follows that the vectors $\mathbf{U}_{\text{ef}}$ and $\mathbf{V}_i$ are also collinear (and codirected).

The convective heat-transfer velocity and the speeds of filtration of phases for the one-dimensional plane-radial flow are also directed along the *r*-axis. Therefore, $\mathbf{V}_i = V_i\mathbf{e}_r$, $\mathbf{U}_{\text{ef}} = U_{\text{ef}}\mathbf{e}_r$ and expression (5) is given in the form

$$U_{\text{ef}}\, \mathbf{e}_r = \mathbf{e}_r \sum_i V_i c_i\, \rho_i / c_{\text{p}}\,. \tag{8}$$

From (8), with the use of the Darcy law, we obtain the expression for the effective convective heat-transfer velocity

$$U_{\text{ef}} = \sum_i V_i c_i\, \rho_i / c_{\text{p}} = -\frac{k}{c_{\text{p}}} \left(\sum_i \frac{c_i\, \rho_i\, f_i (s)}{\mu_i} \right) \frac{dP}{dr}\,. \tag{9}$$

For the gassy liquid flow under consideration at $i = 1, 2, 3$ from (9), taking into account the equality of the speeds of filtration of the oil and the gas dissolved in it, we have

$$U_{\text{ef}} = \frac{(c_1\, \rho_1 + c_2\, \rho_2)\, V_1 + c_3\, \rho_3 V_3}{ms_1\, \rho_1 c_1 + ms_2\, \rho_2 c_2 + ms_3\, \rho_3 c_3 + (1 - m)\, \rho_0 c_0} = -\frac{k}{c_{\text{p}}} \left[(c_1\, \rho_1 + c_2\, \rho_2)\, \frac{f_1 (s)}{\mu_1} + c_3\, \rho_3\, \frac{f_3 (s)}{\mu_3} \right] \frac{dP}{dr}\,.$$

Effective Joule–Thomson and Adiabatic Coefficients. Equating the coefficients of the pressure gradient in Eqs. (1) and (2), we obtain

$$\varepsilon_{\text{ef}} \mathbf{U}_{\text{ef}} = \sum_i \varepsilon_i \mathbf{V}_i c_i\, \rho_i / c_{\text{p}}\,. \tag{10}$$

Substituting into (10) the expression for the convective heat-transfer velocity (4), we have

$$\varepsilon_{\text{ef}} \sum_i \mathbf{V}_i c_i\, \rho_i = \sum_i \varepsilon_i \mathbf{V}_i c_i\, \rho_i\,. \tag{11}$$

Then, multiplying both sides of equality (11) by $\Sigma\, \mathbf{V}_i c_i \rho_i$, we find the expressions for the effective Joule–Thomson coefficient:

$$\varepsilon_{\text{ef}} = \left[\left(\sum_i \varepsilon_i \mathbf{V}_i c_i\, \rho_i \right) \left(\sum_i \mathbf{V}_i c_i\, \rho_i \right) \right] \Big/ \left(\sum_i \mathbf{V}_i c_i\, \rho_i \right)^2 . \tag{12}$$

For a homogeneous porous medium, substituting (5) into (12), we obtain

$$\varepsilon_{\text{ef}} = \sum_i \varepsilon_i V_i c_i \rho_i \Bigg/ \left(\sum_i V_i c_i \rho_i \right). \tag{13}$$

If we express V_i in the form of (6), then (13) is transformed as

$$\varepsilon_{\text{ef}} = \sum_i \varepsilon_i c_i \rho_i f_i(s)/\mu_i \Bigg/ \left(\sum_i c_i \rho_i f_i(s)/\mu_i \right). \tag{14}$$

From expression (14) it follows that the value of the effective Joule–Thomson coefficient does not explicitly depend on the permeability, the pressure gradient, and the total heat capacity per unit volume of the porous medium. Moreover, formula (4) is written independent of the chosen system of coordinates.

For the oil flow under consideration and the gas dissolved in it we have

$$\varepsilon_{\text{ef}} = \frac{(\varepsilon_1 c_1 \rho_1 + \varepsilon_2 c_2 \rho_2) V_1 + \varepsilon_3 V_3 c_3 \rho_3}{(c_1 \rho_1 + c_2 \rho_2) V_1 + V_3 c_3 \rho_3} .$$

The heat capacity per unit volume of the oil pool is determined by the relation

$$c_{\text{p}} = ms_1 \rho_1 c_1 + ms_2 \rho_2 c_2 + ms_3 \rho_3 c_3 + (1 - m) \rho_0 c_0 . \tag{15}$$

In a similar manner, we represent the corresponding relation for the adiabatic coefficient in the form

$$\eta_{\text{ef}} = \frac{s_1 \rho_1 c_1 \eta_1 + s_2 \rho_2 c_2 \eta_2 + s_3 \rho_3 c_3 \eta_3}{s_1 \rho_1 c_1 + s_2 \rho_2 c_2 + s_3 \rho_3 c_3} .$$

Solution of the Hydrodynamic Problem and Construction of Curves for Effective Coefficients. The process of obtaining a solution of the hydrodynamic problem [7] consists of several stages. First, using the continuity equations and the Henry and Darcy laws and taking into account the equality of the pressure gradients in the gas and liquid phases, we find the ratio between the phase permeability functions of the free gas and the carrier phase:

$$\frac{f_3(s)}{f_1(s)} = \frac{\rho_1 \mu_3}{\rho_3 \mu_1} \left(\frac{1}{1 - \alpha P_{\text{s}}} - \frac{1}{1 - \alpha P} \right). \tag{16}$$

One of the phase permeability functions, depending on the gas saturation s, is determined either experimentally or on the basis of the solution of additional theoretical problems

$$f_1 = f_1(s) . \tag{17}$$

Equalities (16) and (17) represent the parametric dependence (s-parameter) of the relative phase permeability of the liquid f_1 or the gas f_3 as a function of the pressure P. This parametric dependence makes it possible to plot the solution of the hydrodynamic problem in the general form for any function $f_1(s)$.

In a particular case, for a porous medium represented by a system of cylindrical pipes of equal diameter, in which the gas bubbles in the liquid regularly alternate, the expression for the phase permeability of the liquid is of the form

$$f_1(s(P)) = \begin{cases} [1 + \xi_1 \xi_2]^{-1}, & P < P_{\text{s}}; \\ 1, & P > P_{\text{s}}, \end{cases}$$

where $\xi_1 = \dfrac{\rho_1 \mu_3}{\rho_3 \mu_1}$ and $\xi_2 = \dfrac{1}{1 - \alpha P_{\text{s}}} - \dfrac{1}{1 - \alpha P}$. In this case, the relations for the oil and gas saturation are functions of the pressure:

$$s_1 = s_2 = (1 + \xi_2\, \rho_1/\rho_3)^{-1}\,, \quad s_3 = s = (1 + \xi_2^{-1}\, \rho_3/\rho_1)^{-1}\,.$$

Then we obtained the expression for the speed of filtration of the liquid in the zones with the free gas in the absence of degassing:

$$V_1 = \begin{cases} -k\left(P_c - P_s - \displaystyle\int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1\, \xi'}\right)\Big/ \mu_1 r \ln (R_c/r_w)\,, & P_w < P_s\,; \\[2ex] -\dfrac{k}{\mu_1}\, \dfrac{P_c - P_w}{\ln (R_c/r_w)}\, \dfrac{1}{r}\,, & P_s \le P_w < P_c\,, \end{cases}$$

where P' is the integration variable; $\xi' = \dfrac{1}{1 - \alpha P_s} - \dfrac{1}{1 - \alpha P'}$.

The expressions for the phase permeability and the speed of filtration of the liquid with the dissolved gas make it possible to construct the relation for the pressure gradient:

$$\frac{dP}{dr} = \begin{cases} (1 + \xi_1\xi_2)\left(P_c - P_s - \displaystyle\int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1\xi'}\right)\Big/\left(r \ln \dfrac{R_c}{r_w}\right), & P < P_s\,; \\[3ex] \dfrac{P_c - P_s}{\ln \dfrac{R_c}{r_s}}\, \dfrac{1}{r} = \left(P_c - P_s - \displaystyle\int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1\xi'}\right)\Big/\left(r \ln \dfrac{R_c}{r_w}\right), & P > P_s\,. \end{cases} \tag{18}$$

The dependence of the pressure P on the radial coordinate r is given by the inverse function

$$r = r_s \exp\left[\ln \frac{R_c}{r_w} \int_{P_s}^{P} \frac{dP'}{1 + \xi_1\xi'} \Big/ \left(P_c - P_s - \int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1\xi'}\right)\right], \quad r < r_s\,, \quad P < P_s\,, \quad P_w < P_s\,; \tag{19}$$

$$r = r_s \exp\left[(P - P_s) \ln \frac{R_c}{r_w} \Big/ \left(P_c - P_s - \int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1\xi'}\right)\right], \quad r > r_s\,, \quad P > P_s\,, \quad P_w < P_s\,. \tag{20}$$

The value of the zone radius r_s in which gas release begins is determined by the expression

$$r_s = R_c \exp\left[(P_c - P_s) \ln \frac{r_w}{R_c} \Big/ \left(P_c - P_s - \int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1\xi'}\right)\right], \quad P_w < P_s\,.$$

In solving the hydrodynamic problem, we found the density fields of the free gas sources:

$$q = \begin{cases} \dfrac{\rho_1 \alpha\, (1 + \xi_1\xi_2)}{r^2\, (1 - \alpha P)^2 \ln^2 (R_c/r_w)}\, \dfrac{k}{\mu_1} \left(P_c - P_s - \displaystyle\int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1\xi'}\right)^2, & P < P_s\,; \\[3ex] 0\,, & P > P_s\,. \end{cases} \tag{21}$$

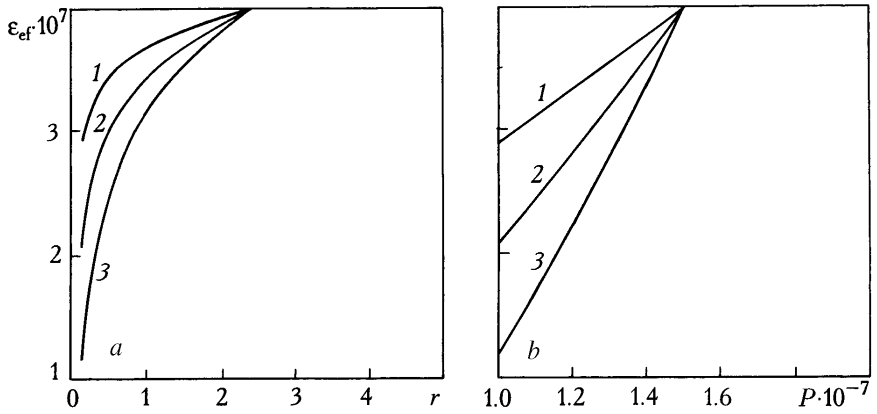

Fig. 2. Effective Joule–Thomson coefficient as a function of the radius (a) and pressure (b) for various values of the gas solubility: 1) $\alpha = 4\cdot 10^{-9}$ 1/Pa; 2) $7\cdot 10^{-9}$; 3) $1\cdot 10^{-8}$.

Expressions for the Effective Joule–Thomson Coefficient, the Effective Adiabatic Coefficient, the Velocity of Convective Heat Transfer, and the Heat Conductivity per Unit Volume upon Filtration of Gassy Oil. The relation for ε_{ef}, taking into account the solution of the hydrodynamic problem and the Henry law, is of the form

$$\varepsilon_{ef} = \frac{\varepsilon_1 c_1 + \varepsilon_2 c_2 \xi_3 + \varepsilon_3 c_3 \xi_2}{c_1 + c_2 \xi_3 + c_3 \xi_2}, \quad \text{where} \quad \xi_3 = \frac{\alpha P}{1 - \alpha P}.$$

Since at pressures higher than the saturation pressure $P > P_s$ the degassing is absent, $\xi_3(P_s) = \alpha P_s/(1 - \alpha P_s) = \xi_4$, $\xi_2(P_s) = 0$,

$$\varepsilon_{ef} = \begin{cases} \dfrac{\varepsilon_1 c_1 + \varepsilon_2 c_2 \xi_3 + \varepsilon_3 c_3 \xi_2}{c_1 + c_2 \xi_3 + c_3 \xi_2}, & P < P_s; \\[2ex] \dfrac{\varepsilon_1 c_1 + \varepsilon_2 c_2 \xi_4}{c_1 + c_2 \xi_4}, & P > P_s. \end{cases} \tag{22}$$

Formula (22) shows the pressure dependence of the effective Joule–Thomson coefficient. Its relation to the radius is given parametrically by the addition to (22) of the expression for the pressure distribution obtained from (18) in implicit form:

$$\int_{P_s}^{P} \frac{dP'}{1 + \xi_1 \xi'} = \left(P_c - P_s - \int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1 \xi'} \right) \ln \frac{r}{r_s} \Big/ \ln \frac{R_c}{r_w}, \quad r < r_s, \quad P < P_s, \quad P_w < P_s; \tag{23}$$

$$P = P_s + \left(P_c - P_s - \int_{P_s}^{P_w} \frac{dP'}{1 + \xi_1 \xi'} \right) \ln \frac{r}{r_s} \Big/ \ln \frac{R_c}{r_w}, \quad r > r_s, \quad P > P_s, \quad P_w < P_s. \tag{24}$$

In calculating the radius dependence of ε_{ef}, the pressure P playing the role of the parameter is excluded from (22).

Figure 2 shows the dependence of ε_{ef} on the distance to the well center and the pressure. It is seen that ε_{ef} in the zone of the absence of degassing $r_s < r < R_c$, $P_s < P < P_c$ remains constant and equal to ε of the liquid (oil). In the degassing zone of $r_w < r < R_c$, $P_w < P < P_s$, the effective Joule–Thomson coefficient increases with increasing radius (Fig. 2a) and pressure (Fig. 2b).

At the same time, the calculations correspond to the liquid (oil) filtration from the pool to the well, i.e., toward lower r. Observation of the movement of a small volume of discharged liquid reveals the following order of

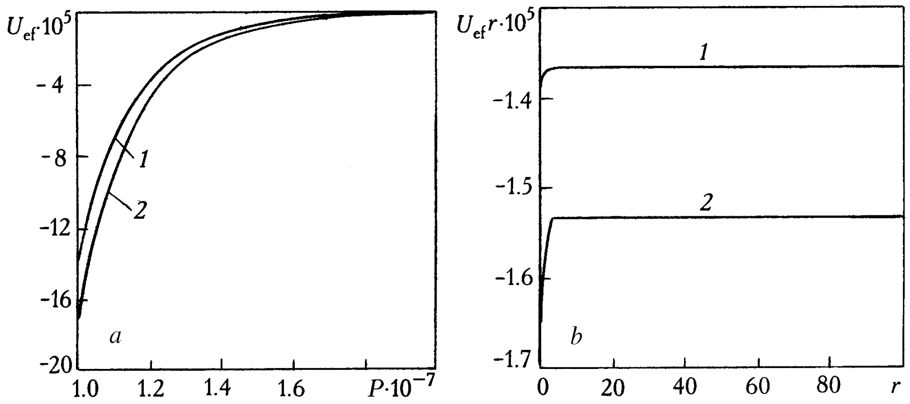

Fig. 3. Relation of the effective convective heat-transfer velocity to the pressure (a) and dependence of the product $U_{ef}r$ on the distance to the well center (b) for various values of the gas solubility: 1) $\alpha = 1\cdot 10^{-9}$ 1/Pa; 2) $1\cdot 10^{-8}$.

change in ε_{ef}: in the zone of the absence of degassing, ε_{ef} remains constant; when the discharged volume reaches the saturation pressure $P_s(r = r_s)$, the value of ε_{ef} begins to decrease in the course of movement, and with increasing solubility of gas α in the degassing zone ε_{ef} decreases. Calculations were made for the following values of the parameters: $c_0 = 800$ J/(K·kg), $c_1 = c_2 = 200$ J/(K·kg), $c_3 = 2400$ J/(K·kg), $r_w = 0.1$ m, $R_c = 100$ m, $P_c = 2\cdot 10^7$ Pa, $P_w = 1\cdot 10^7$ Pa, and $P_s = 1.5\cdot 10^7$ Pa. It should be noted that in expression (22) $\varepsilon_1 = \varepsilon_2 = 4\cdot 10^{-7}$ K/Pa, and ε_3 was found from the familiar thermodynamic relation on the basis of the equation of state of the real gas.

With the above parameters corresponding to the most widely distributed conditions ε_{ef} does not change its sign, remaining in the range of positive values. This is explained by the considerable difference in the mass content of the carrier phase and the free gas that compensates with excess for the almost tenfold excess of the modulus of the Joule–Thomson coefficients of gas and oil.

The heat capacity per unit volume of the pool upon gassy oil filtration is determined according to (15), taking into account the found expressions for the oil and gas saturation:

$$c_p = \begin{cases} m\rho_1 (1 + \xi_2 \rho_1/\rho_3)^{-1} (c_1 + c_2 \xi_3) + m\rho_3 c_3 (1 + \xi_2^{-1} \rho_3/\rho_1)^{-1} + (1 - m) \rho_0 c_0 , & P < P_s ; \\ m\rho_1 (c_1 + c_2 \xi_4) + (1 - m) \rho_0 c_0 , & P > P_s . \end{cases} \tag{25}$$

The effective adiabatic coefficient η_{ef} and the convective transfer velocity U_{ef} in the case of gassy oil filtration for the degassing zone ($P < P_s$) and in the absence of gas liberation ($P > P_s$) from the carrier phase are of the form

$$\eta_{ef} = \begin{cases} \dfrac{c_1\eta_1 + c_2\eta_2\xi_3 + c_3\eta_3\xi_2}{c_1 + c_2 \xi_3 + c_3 \xi_2} , & P < P_s ; \\ \dfrac{c_1\eta_1 + c_2\eta_2 \xi_4}{c_1 + c_2 \xi_4} , & P > P_s . \end{cases} \tag{26}$$

$$U_{ef} = \begin{cases} \dfrac{\rho_1 V_1 (c_1 + c_2 \xi_3 + c_3 \xi_2)}{c_p} , & P < P_s ; \\ \dfrac{\rho_1 V_1 (c_1 + c_2 \xi_4)}{c_p} , & P > P_s . \end{cases} \tag{27}$$

The dependences of η_{ef} and U_{ef} on the radial coordinate r are given parametrically by adding to (26)–(27) the expression for the pressure distribution (23)–(24).

Figure 3a shows the convective heat-transfer velocity as a function of pressure for various solubility coefficients of the gas. It is seen that with increasing pressure the effective velocity of convective heat transfer increases; as the solubility coefficient α increases, there is a slight increase in the modulus of the convective heat-transfer velocity U_{ef}.

Figure 3b gives the curves showing the product of the convective heat-transfer velocity by the radius $U_{\mathrm{ef}}r$ as a function of the coordinate for various solubility coefficients of the gas. It is seen that to an accuracy from 0.7% for $\alpha = 10^{-9}$ Pa^{-1} to 7% for $\alpha = 1 \cdot 10^{-8}$ Pa^{-1} the product $U_{\mathrm{ef}}r$ is independent of r.

Apparent Joule–Thomson Coefficient. The equation of the barothermal effect energy (3) in the case of gassy liquid filtration in the cylindrical system of coordinates for the one-dimensional case, with the heat conductivity neglected, is of the form

$$\frac{\partial T}{\partial t} + U_{\mathrm{ef}}\left(\frac{\partial T}{\partial r} + \varepsilon_{\mathrm{ef}}\frac{\partial P}{\partial r}\right) - \eta_{\mathrm{a}}\,\Pi\,\frac{\partial P}{\partial t} + \frac{Lq}{c_{\mathrm{p}}} = 0 \,. \tag{28}$$

Assuming the pressure gradient and the convective heat-transfer velocity other than zero, $\partial P/\partial \neq 0$ and $U_{\mathrm{ef}} \neq 0$, expression (28) can be given in the form

$$\frac{\partial T}{\partial t} + U_{\mathrm{ef}}\left[\frac{\partial T}{\partial r} + \left(\varepsilon_{\mathrm{ef}} + \frac{Lq}{c_{\mathrm{p}}\dfrac{\partial P}{\partial r}U_{\mathrm{ef}}}\right)\frac{\partial P}{\partial r}\right] - \eta_{\mathrm{a}}\,\Pi\,\frac{\partial P}{\partial t} = 0 \,.$$

Denote the quantity that represents the addition to the effective Joule–Thomson coefficient and is responsible for the temperature effect of liquid degassing by δ_ε:

$$\delta_\varepsilon = \begin{cases} \dfrac{Lq}{U_{\mathrm{ef}}\dfrac{dP}{dr}c_{\mathrm{p}}}\,, & P < P_{\mathrm{s}}\,; \\ 0\,, & P > P_{\mathrm{s}}\,. \end{cases} \tag{29}$$

Substituting into (29) the expressions for the pressure gradient $\partial P/\partial r$, the densities of gas sources q, the effective convective heat-transfer velocity U_{ef}, and the heat conductivity per unit volume of the gas c_{p} from (18), (21), (25), and (27), we obtain

$$\delta_\varepsilon = \begin{cases} -\dfrac{L\alpha}{(1-\alpha P)^2\,(c_1 + c_2\,\xi_3 + c_3\,\xi_2)}\,, & P < P_{\mathrm{s}}\,; \\ 0\,, & P > P_{\mathrm{s}}\,. \end{cases}$$

Below, by the apparent Joule–Thomson coefficient is meant the physical quantity representing the sum of the effective Joule–Thomson coefficient and the coefficient δ_ε:

$$\varepsilon_{\mathrm{a}} = \varepsilon_{\mathrm{ef}} + \delta_\varepsilon \,.$$

The expression for the apparent Joule–Thomson coefficient for pressures larger or smaller than the saturation pressure is written in the form

$$\varepsilon_{\mathrm{a}} = \begin{cases} \dfrac{\varepsilon_1 c_1 + \varepsilon_2 c_2\,\xi_3 + \varepsilon_3 c_3\,\xi_2 - L\alpha/(1-\alpha P)^2}{c_1 + c_2\,\xi_3 + c_3\,\xi_2}\,, & P < P_{\mathrm{s}}\,; \\ \dfrac{\varepsilon_1 c_1 + \varepsilon_2 c_2\,\xi_4}{c_1 + c_2\,\xi_4}\,, & P > P_{\mathrm{s}}\,. \end{cases}$$

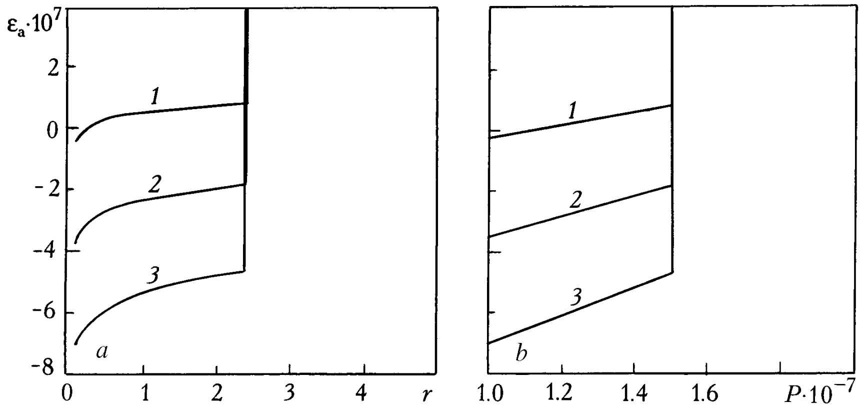

Fig. 4. Apparent Joule–Thomson coefficient as a function of the distance to the well center (a) and pressure (b) for various values of the gas solubility: 1) $\alpha = 4 \cdot 10^{-9}$ 1/Pa; 2) $7 \cdot 10^{-9}$; 3) $1 \cdot 10^{-8}$.

By the apparent adiabatic coefficient η_a is meant the physical value representing the sum of the effective adiabatic coefficient η_{ef} and the coefficient δ_η:

$$\eta_a = \eta_{ef} + \delta_\eta \, .$$

The expressions for δ_η and η_a are given in the form

$$\delta_\eta = \begin{cases} \dfrac{L\alpha}{(1 - \alpha P)^2 \, (c_1 + c_2 \, \xi_3 + c_3 \, \xi_2)} \, , & P < P_s \, ; \\ 0 \, , & P > P_s \, . \end{cases}$$

$$\eta_a = \begin{cases} \dfrac{c_1 \eta_1 + c_2 \eta_2 \, \xi_3 + c_3 \eta_3 \, \xi_2 + L\alpha/(1 - \alpha P)^2}{c_1 + c_2 \, \xi_3 + c_3 \, \xi_2} \, , & P < P_s \, ; \\ \dfrac{\eta_1 c_1 + \eta_2 c_2 \, \xi_4}{c_1 + c_2 \, \xi_4} \, , & P > P_s \, . \end{cases}$$

Figure 4a shows the curves of the dependences of ε_a on the distance to the well center and pressure. From the figures it follows that the apparent Joule–Thomson coefficient in the zone where there is no degassing ($r_s < r < R_c$, $P_s < P < P_c$) is constant and equal to the Joule–Thomson coefficient of the liquid (oil). At the point of $r = r_s$ and $P = P_s$ the apparent Joule–Thomson coefficient changes stepwise, since here the density of sources of phase transitions becomes other than zero. In the degassing zone ($r_w < r < R_c$, $P_w < P < P_s$), the apparent Joule–Thomson coefficient ε_a increases with increasing radius (Fig. 4a) and pressure (Fig. 4b).

At the same time, the calculations correspond to the movement of the liquid from the pool to the well, i.e., toward smaller values of the radial coordinate r. Observation of the movement of a small volume of discharged liquid reveals a certain order of the change in ε_a. In the zone where there are no phase transitions, ε_a remains constant. When the discharged volume reaches the saturation pressure P_s ($r = r_s$), the apparent Joule–Thomson coefficient decreases stepwise and then decreases in the course of movement. It is important to note that the pressure dependence of the apparent Joule–Thomson coefficient is close to linear in the degassing zone (Fig. 4b). The value of the step is strongly dependent on the solubility coefficient α and weakly dependent on the saturation pressure P_s. It increases with increasing α.

It is important to consider the contribution of the value of δ_ε to the value of the apparent Joule–Thomson coefficient ε_a. Figure 5a shows the dependence of the coefficient δ_ε on the distance to the well center. Note that in the

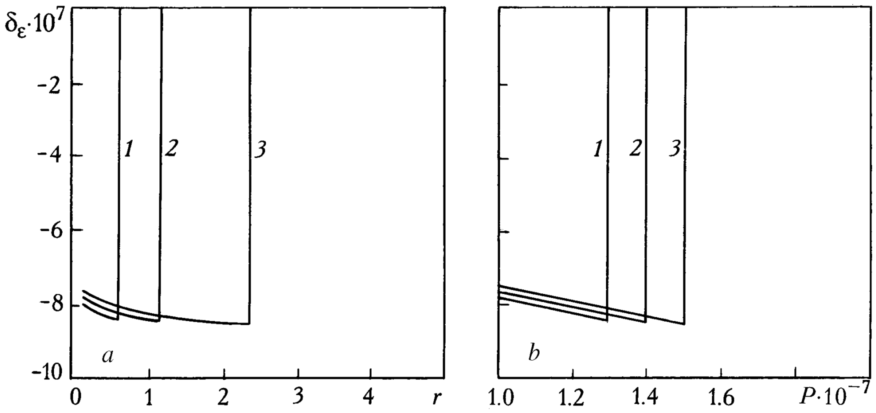

Fig. 5. Coefficient δ_ε as a function of the distance to the well center (a) and pressure (b) for various values of the saturation pressure: 1) $P_s = 1.3 \cdot 10^7$ Pa; 2) $1.4 \cdot 10^7$; 3) $1.5 \cdot 10^7$; $\alpha = 1 \cdot 10^{-8}$ 1/Pa.

case under consideration the coefficient δ_ε is negative, since the degassing is accompanied by heat absorption. From Fig. 5a it is seen that in the zone with no degassing $r_w < r < r_s$ with increasing radius r the value of δ_ε decreases; at the point of $r = r_s$ the value of δ_ε increases stepwise to zero and then remains constant. With increasing saturation pressure P_s, the step of the coefficient δ_ε is shifted into the region of large values of the radius r and the value of the step increases. In the case under consideration, $\delta_\varepsilon = -8 \cdot 10^{-7}$ K/Pa, which is twice as large in modulus as the Joule–Thomson coefficient for oil. Figure 5b shows the plots of the pressure dependence of the coefficient δ_ε for various values of the saturation pressure. From this figure it is seen that δ_ε decreases (the modulus of δ_ε increases) in the degassing zone ($P_w < P < P_s$) with increasing distance to the well axis and is equal to zero in the zone of the absence of phase transitions $P_s < P < P_c$. The coefficient δ_ε is linearly dependent on the pressure.

CONCLUSIONS

1. With increasing gas solubility α and saturation pressure P_s, all other parameters being constant, there is a decrease in ε_{ef} in the degassing zone, which is explained by the increase in the free gas content having a negative Joule–Thomson coefficient. It is important to note that with the above parameters corresponding to the most widely distributed conditions ε_{ef} does not change its sign, remaining in the range of positive values.

2. The value of the apparent Joule–Thomson coefficient ε_a and coefficient δ_ε changes stepwise at the point of onset of degassing ($P = P_s$, $r = r_s$). The value of the step is strongly dependent on the gas solubility coefficient α and weakly dependent on the saturation pressure P_s. At large enough values of the gas solubility coefficient ε_a changes its sign. Outside the degassing zone the value of ε_a remains constant and equal to ε_{ef}.

3. With increasing pressure P and radius r in the degassing region ε_a increases (the absolute value of ε_a decreases), and the pressure dependence to a high accuracy thereby can be considered to be linear. Such behavior of ε_a coincides with the dependence of the effective Joule–Thomson coefficient on the radius r and pressure P. Despite the fact that the value of ε_a largely depends on the contribution of the degassing at large enough solubilities of the gas α, the dependence of ε_a on the pressure P and the coordinate r are determined by the gas-phase filtration.

4. In the practically important range of parameters, the product of the effective convective heat-transfer velocity by the radial coordinate $U_{ef}r$ is practically independent of the distance to the well axis, since the corresponding change does not exceed 7%.

NOTATION

a, constant of the Van der Waals equation of state, m^6/(Pa·mole^2); α_p, heat-conductivity coefficient of the pool, m^2/sec; b, constant of the Van der Waals equation of state, m^3/mole; c, heat capacity, J/(K·kg); c_p, heat capacity per unit volume of the pool, J/(K·m^3); $\mathbf{e}_r$, unit vector of the cylindrical system of coordinates; $f(s)$, phase permeability; k, permeability, m^2; L, phase-transition heat, J/kg; m, porosity; q, density function of degassing forces (phase-transition intensity), kg/(m^3·sec); P, pressure, Pa; P_c, pool pressure, Pa; P_s, saturation pressure, Pa; P_w, well pressure, Pa; r, distance to the well center, m; r_w, well radius, m; r_s, radius of the saturation zone, m; R, universal gas constant, J/(mole·K); M, molar mass, kg/mole; R_c, feed contour, m; s, gas saturation; s_i, saturation of the ith phase (ith phase); T, temperature (temperature difference), K; W, specific internal energy, J/kg; α, gas solubility coefficient, 1/Pa; λ, heat-conductivity coefficient, W/(m·K); ε, Joule–Thomson coefficient; K/Pa; μ, viscosity, Pa·sec; η, adiabatic coefficient, K/Pa; θ^*, interphase heat-exchange coefficient, W/(K·m^3); $\mathbf{V}$, filtration velocity, m/sec; V, component of the velocity vector, m/sec; ρ, density, kg·m^3. Subscripts: 0, pertains to the porous medium skeleton; i, j, number of the medium phase (component); p, pool; a, apparent; c, contour; ef, effective; s, saturation; w, well.

REFERENCES

1. M. D. Rozenberg, S. A. Kundin, A. K. Kurbanov, N. I. Suvorov, and G. Yu. Shovkrinskii, *Filtration of Aerated Fluid and Other Multicomponent Mixtures in Oil Pools* [in Russian], Moscow (1969).
2. L. D. Landau and E. M. Lifshits, *Statistical Physics* [in Russian], Moscow (1964).
3. A. I. Philippov, E. M. Devyatkin, and S. A. Filippov, in: *Proc. Int. Conf. on Dynamics of Multiphase Systems*, ICMS’2000, June 15–17, 2000, Ufa, Russia (2000), pp. 444–451.
4. É. B. Chekalyuk, *Thermodynamics of an Oil Pool* [in Russian], Moscow (1965).
5. A. I. Filippov, *Well Thermometry of Transient Processes* [in Russian], Saratov (1990).
6. A. I. Filippov and E. M. Devyatkin, *Teplofiz. Vys. Temp.*, No. 2, 274–282 (2001).
7. A. I. Filippov and E. M. Devyatkin, in: A. I. Filippov (ed.), *Problems of Applied Thermophysics* [in Russian], Interuniversity Collection of Sci. Papers, Sterlitamak (2000), pp. 42–61.